If V is sufficiently high (supersonic flow), and c_p sufficiently low, this temperature drop may become very significant. Assume now that the temperature recovery factor on the walls of the rotating channels is equal to that of a flat plate (this seems acceptable as a first approximation if the turning in the channel is very gradual); then

$$T_{aw} = T + r(T_{02} - T) \tag{7}$$

In a laminar boundary layer the recovery factor is given by

$$r \cong Pr^{1/2} \qquad Pr = c_p\mu/k \tag{8}$$

Let us analyze the characteristics of a number of working fluids proposed to date for space dynamic conversion systems, the most significant characteristics of which are listed in Table 1, which was compiled from Refs. 1–6; methane and ethane were added to the list, because of their very low Prandtl numbers in the range of temperatures in which we are interested.

Equations (6) and (7) are solved with these data and plotted in Figs. 1 and 2, respectively. It may be noted that for the rare gases, the assumption of constant γ and Pr is in very good agreement with experiments.

Before analyzing the results obtained so far, let us discuss another problem particularly important in Brayton-cycle turbo-generators: heat exchangers and radiators weight minimization. Power unit configurations, including heat-source heat exchanger, heat-sink heat exchanger, recuperator, and radiator, have been proposed.[5]

The favorable effect of low Prandtl number on heat-transfer rates is qualitatively shown through the Reynolds analogy, which allows one to deduce heat-transfer rates from knowledge of skin friction.

The heat transferred per unit area and per unit time is

$$q = St\rho_e V_e c_p(T_w - T_{aw}) \tag{9}$$

$$St = C_f Pr^{-2/3}/2 \tag{10}$$

where St is the Stanton number, ρ_e, V_e are the density and the velocity at the boundary-layer edge, respectively, T_w is the actual wall temperature, and C_f is the skin-friction coefficient. Equation (10) holds both for laminar and turbulent boundary layers. The foregoing results are not as simple as they first may appear, the skin-friction coefficient being dependent upon Reynolds number, Mach number, wall, and freestream temperature, and, finally, on the relationship between viscosity and temperature.[7]

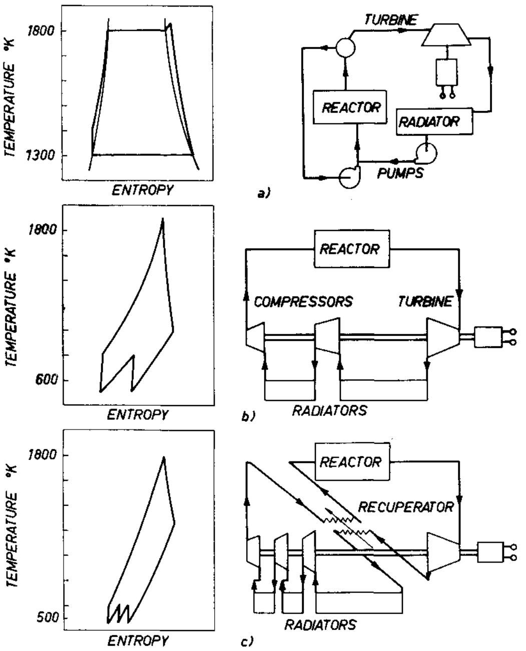

Fig. 3 Some possible configurations of turbo-converters cycles, qualified for space applications.

The trend shown in Eqs. (9) and (10) is confirmed by detailed computations, in one of which the surface of a tabular counterflow recuperator was found to be given by the expression[5]

$$S = \lambda Pr\mathfrak{M}^{1/2}(1 + r_c^{-1/2})^2(\gamma - 1)/\gamma p_3 \tag{11}$$

where λ is a coefficient independent of fluid properties, p_3 is the pressure at the compressor inlet, and r_c is the compressor pressure ratio.

According to Eq. (11), the best performance with respect to the reduction of the recuperator weight appears to be given by methane, due to its low Pr at temperatures of practical interest and to its low $\mathfrak{M}$.

If experimental results confirm the possibility of raising the maximum gas or vapor temperature with respect to current values, this will result in improved turbo-converters efficiency and reduced radiator weight.

Figure 3 shows some possible future configurations of space dynamic power systems, employing both the Rankine and the Brayton cycles.

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Flame Spread on Solid Propellant

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Nomenclature

c = solid heat capacity, cal/g°C
f = surface heat flux; f_0, heat flux at edge of burning zone, cal/cm²-sec
F = function defined by Eq. (2), determined by Eq. (1)
k = thermal conductivity, cal/cm-sec-°C
p = pressure, atm
s = intermediate time variable, sec
t = time: t_0, ignition time at $x = 0$; t_i, ignition time at position x, with time zero set at the start of gas flow; sec
T = surface temperature rise above T_0: T_0, initial slab temperature; T_i, value of T at ignition; °C

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u = gas velocity, m/sec
x = downstream distance from flame front at time zero, cm
y = auxiliary position variable, distance from x to flame position at time s, cm
α = parameter to nondimensionalize distance [Eq. (1)], cm^{-1}
ρ = density, g/cm^3
τ = dimensionless time variable = βt
ψ = heat-flux decay function defined by $f = f_0\psi(\alpha y)$

Introduction

An igniter in a large solid-propellant engine usually ignites directly only a part of the exposed surface area of the grain which then provides the energy needed to ignite the remaining surface. This note presents the results of a study of flow-assisted flame spread over the freshly cut surface of a composite propellant, with conditions of constant pressure and gas velocity. Although p and u are not constant during the ignition transient of an actual motor, the constant process condition case was chosen for this study in an attempt to learn of the influence of these variables upon flame spread.

Experimental Technique

The rarefaction tube was found to be a convenient tool for producing controlled conditions of constant p and u. A tube of circular cross section, closed at one end and fitted with a nozzle at the other, was initially pressurized with a diaphragm over the nozzle. The T-shaped propellant samples were mounted in a slot milled in the top surface of a streamlined plate, which was located near the nozzle end of the tube. The propellant was ignited across the upstream width with a hot wire, a time interval was allowed to permit the propellant to reach steady burning, and the diaphragm was broken to start the rarefaction. The test pressure and gas velocity were determined by the initial pressure and the nozzle-to-tube area ratio. The progress of flame spread was observed by means of high-speed motion pictures taken through a transparent tube section.

Auxiliary measurements of $T(t)$ were made with thin-film platinum resistance thermometers bonded to fired pyrophilite bases mounted just downstream from the propellant slab.[3] The $T(t)$ oscilloscope traces were compared with theoretical $T(t)$ curves.

Theory

The theory[3] from which selected results are given below pictures the flow, at constant p and u, of hot combustion products across a flat, unignited surface. The following assumptions are made: 1) heat flow into the propellant is one-dimensional, perpendicular to the surface; 2) the propellant slab is a semi-infinite body of constant k, ρ, and c; 3) ignition occurs when the surface temperature T reaches a T_i characteristic of the propellant; 4) given p and u, the heat flux is a function only of the distance of the point of interest from the flame front.

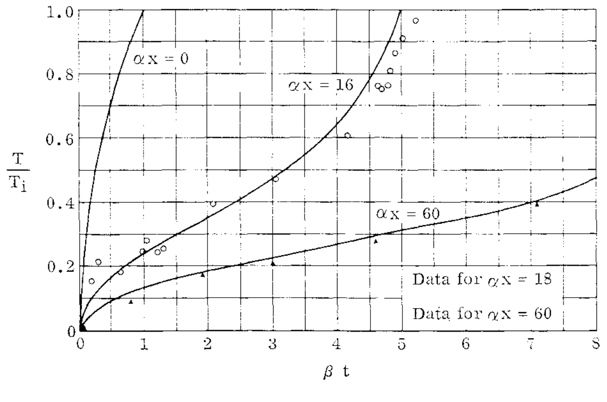

Fig. 1 Theoretical temperature-time curves, in dimensionless form, with examples of experimental data.

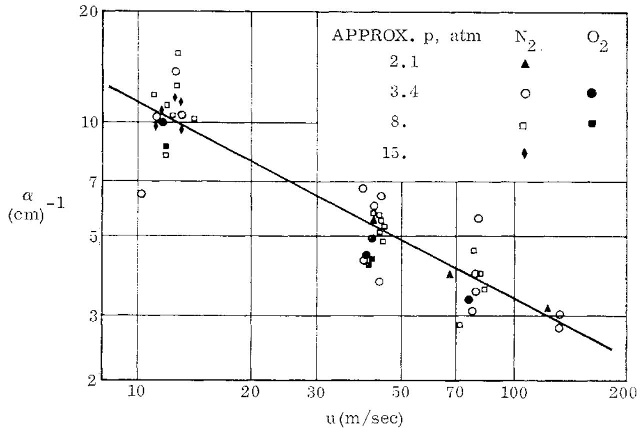

Fig. 2 Correlation of reciprocal distance parameter α with gas velocity.

The one-dimensional, unsteady-state heat-transfer equation for a semi-infinite solid is solved with appropriate boundary conditions for the surface temperature of the propellant slab model. The result, after introduction of the general form of the heat-flux decay function discussed in Ref. 3, is

$$T_i = f_0(\pi k\rho c)^{-1/2}\int_0^{t_i}(t_i - s)^{-1/2}\psi(\alpha y)ds \qquad (1)$$

where α is a reciprocal length parameter, which nondimensionalizes the distances. If we assign a suitable specific form to $\psi(\alpha y)$ and utilize the fact that $(x - y)$ is the same function of s as x is of t_i, we can, in principle, extract a unique $x(t_i)$ relationship from Eq. (1). In dimensionless form, that is not committed to values of T_i, f_0, or $k\rho c$ but is committed to the form of $\psi(\alpha y)$, we can get

$$\alpha x = F(\tau) \equiv F(\beta t) \qquad (2)$$

where β, the nondimensionalizing parameter for time, is defined as $\beta = 4\, f_0^2/\pi^{1/2}(k\rho c)T_i^2$. Given an explicit form of Eq. (2), the flame speed can be calculated from $dx/dt = (\beta/\alpha)\, d(\alpha x)/d\tau$.

An approximate analysis of heat transfer from gas to solid through a laminar boundary layer led to the choice of the following form for $\psi(\alpha y)$:

$$\psi(\alpha y) = (1 + \alpha y)^{-n} \qquad (3)$$

The choice of the exponent n was made on empirical grounds, comparing experimental and predicted temperature histories, as represented in Fig. 1 for $n = \frac{1}{2}$.

One additional observation is to be made. If we take $\alpha x = 0$, that is, we direct attention to the part of the surface adjacent to the initial flame front at $s = 0+$, we find (neglecting the slow, unassisted flame spread) that a finite time t_0 is needed to start the flame spread. All suitable forms of $\psi(\alpha y)$ must give $\psi(0) = 1$. The theory yields $\tau = 1$ for $x = 0$, or $4f_0^2t_0/\pi k\rho c T_i^2 = 1$, which is one of the basic relationships employed by Baer[1] and McCune[2] to interpret convective flux ignition data. It provides a bridge between this study and other information about the ignition of the same propellant.

Results and Discussion

The exponent n for Eq. (3) was found empirically from the flame spread data to be $\frac{1}{2}$. This result was confirmed by a plot of experimental $T(t)$ data in Fig. 1, where agreement with theoretical curves is satisfactory.

The observed rates of flame spread, dx/dt_i, ranged from 13 cm/sec at the beginning of flame spread for the lowest p and u to about 400 cm/sec after the flame had spread 5 cm and for higher p and u. These extremes represent, respectively, rates about 40 and 400 times the normal burning

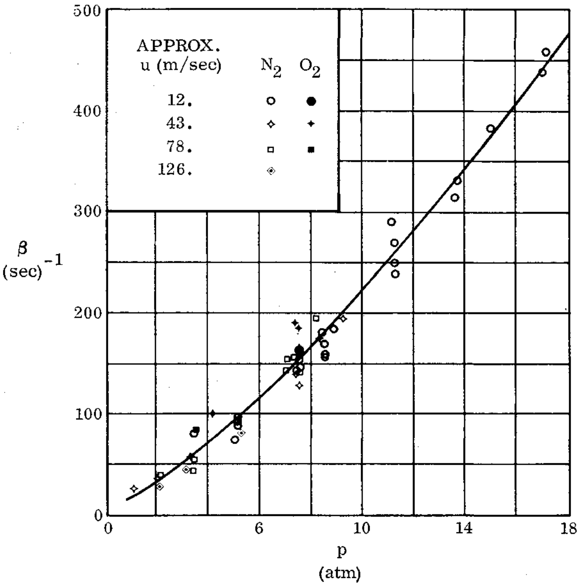

Fig. 3 Correlation of reciprocal time parameter β with pressure.

rates at the same pressures. The burning zone advanced by the appearance of discrete groups of tiny flames, rather than by the movement of a continuous flame front. Tiny "secondary ignition spots" appeared somewhat in advance of the main flame front and spread at a slower rate than the advance of the main front, so that they were usually overtaken by and consolidated with the main burning area before spreading to an appreciable extent. Although the flame spread in small, discrete jumps, the position-time points formed reasonably smooth curves when plotted.

Values of α and β, obtained from the flame-spread data by a curve-matching technique, are presented in Figs. 2 and 3, which show that α is a strong function of gas velocity ($\alpha u^{0.55} = 41.3$) but apparently not of pressure, and β increases with approximately the 1.25 power of pressure but is not affected significantly by velocity.

The composition of β is known from its definition. The increase in f_0/T_i with increasing pressure is explained qualitatively by the increase in density of the hot gas from the burning zone. Apparently, for transport of hot gas from the burning zone to a contiguous unignited surface, all of the velocities studied here (>10 m/sec) are equally effective, i.e., transport effectiveness is complete over a wide range of velocities, including the range studied and extending to a velocity sufficient to extinguish the flame.

The inverse relationship between α and u is to be expected from boundary-layer considerations. The empirical origin of α, however, makes it difficult to comment on the degree of velocity dependence and the absence of pressure dependence. Two effects of increasing velocity are easily conceived: heat transfer is enhanced; and, at a given x, the hot gas has less time to radiate or otherwise lose its energy to the surroundings.

Several runs were made with oxygen as the tube gas in place of nitrogen (Figs. 2 and 3). Since no appreciable consistent effect was found, it is concluded that, except to provide the aerodynamic environment, the tube gas did not participate significantly.

Conclusions

In rocket-design practice, we are confronted with the problem of anticipating, perhaps of regulating, the progress of ignition over large areas of propellant surface under conditions of changing pressure, energy distribution, and internal flow patterns. Flame spread is the concluding phase of the complex sequence of events that comprises the over-all ignition transient. It is, therefore, the one most susceptible to influence by igniter behavior, propellant response, geometry, and scale, insofar as these affect the time-dependent heat-flux distribution. From laboratory experiments conducted with constant process conditions, therefore, the principal products expected are clues to the influence of the process conditions. The analysis of this study produces two flame-spread parameters in which, fortuitously, the pressure and velocity effects are isolated. The influence of pressure appears to be greater than the influence of velocity.

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Further Comments on Correlations for Shifting Rocket Thrust

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PREVIOUSLY mentioned[1] were the empirical correlations between 1) theoretical enthalpy drops in rocket-combustion products on equilibrium expansion from 1000 psia to 1 atm and 2) heats of reaction of the propellants at 298°K. With the exception of a few anomalies,[2] they apply when all products are gaseous and the systems are arranged with respect to the average number of atoms per mole of major (i.e., "effective") species, regardless of dissociations or interreactions. This note presents correlations that hold when the effective species are not all gases so the original correlations are not applicable. It also comments on choosing effective species for the correlations and on the subordinate nature of rocket temperatures.

Table 1 Adjusted ΔH_{f298} for correlations with some nongaseous products

Atoms/mole	Species	Accepted ΔH_{f298}, kcal/mole	Adjustment, kcal/mole	Adjusted ΔH_{f298}, kcal/mole
2	BeO(l)	−129.1	0.0[a]	−129.1
	BeO(s)	−143.1	+1.0	−142.1
	BN(s)	−60.3	−3.0	−63.3
	LiF(l)	−140.7	+7.0	−133.7
	LiF(s)	−145.1	+3.0	−142.1
	MgO(s)	−143.7	+3.0	−140.7
3	ZrO_2(l)	−245.5	0.0[a]	−245.5
	ZrO_2(s)	−261.5	−4.0	−265.5
	Li_2O(l)	−132.3	+5.0	−127.3
	Li_2O(s)	−142.6	−0.5	−143.1
	SiO_2(l)	−209.4	−3.5	−212.9
5	Al_2O_3(l)	−356.5	0.0[a]	−356.5
	Al_2O_3(s)	−400.4	+9.0	−391.4

[a] No adjustment, reference species.

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